Computational Hydraulics

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Review of Hydraulics of Pipe Flows

Module2

3 lectures

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General Introduction

- Pipe flows are mainly due to pressure difference between two sections
- Here also the total head is made up of pressure head, datum head and velocity head
- The principle of continuity, energy, momentum is also used in this type of flow.
- For example, to design a pipe, we use the continuity and energy equations to obtain the required pipe diameter
- Then applying the momentum equation, we get the forces acting on bends for a given discharge

General introduction

- In the design and operation of a pipeline, the main considerations are head losses, forces and stresses acting on the pipe material, and discharge.
- Head loss for a given discharge relates to flow efficiency; i.e an optimum size of pipe will yield the least overall cost of installation and operation for the desired discharge.
- Choosing a small pipe results in low initial costs, however, subsequent costs may be excessively large because of high energy cost from large head losses

Energy equation

- The design of conduit should be such that it needs least cost for a given discharge
- The hydraulic aspect of the problem require applying the one dimensional steady flow form of the energy equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Where p/γ =pressure head

$\alpha V^2/2g$ =velocity head

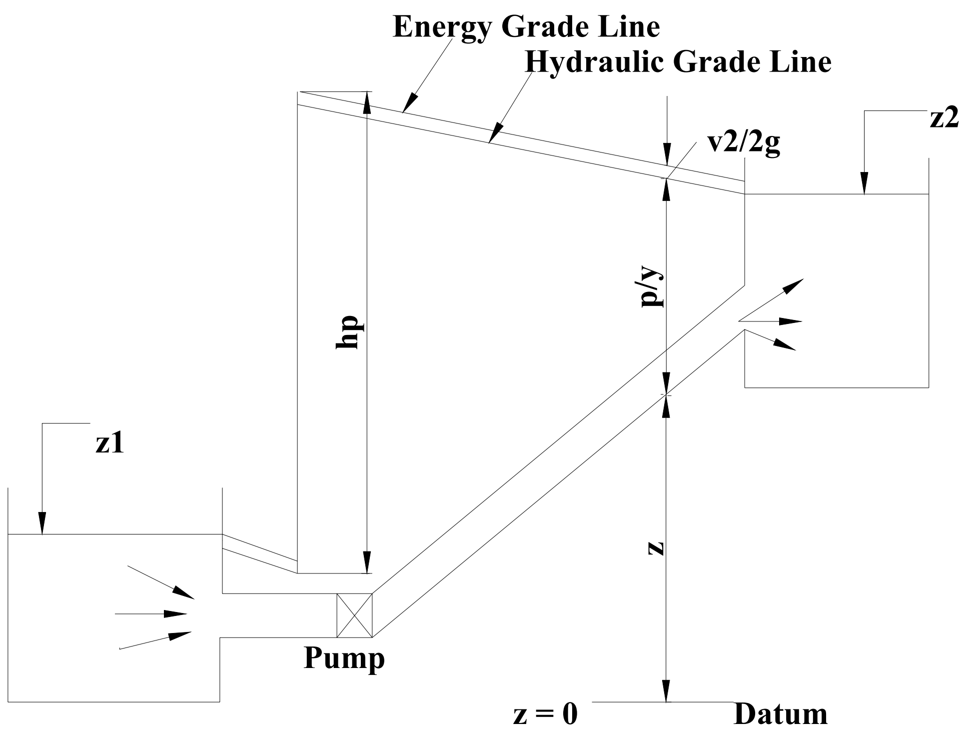
z =elevation head

h_p=head supplied by a pump

h_t =head supplied to a turbine

h_L =head loss between 1 and 2

Energy equation



The Schematic representation of the energy equation

Energy equation

Velocity head

- In $\alpha V^2/2g$, the velocity V is the mean velocity in the conduit at a given section and is obtained by $V=Q/A$, where Q is the discharge, and A is the cross-sectional area of the conduit.
- The kinetic energy correction factor is given by α, and it is defines as, where u=velocity at any point in the section

$$\alpha = \frac{\int_A u^3 dA}{V^3 A}$$

- α has minimum value of unity when the velocity is uniform across the section

Energy equation cont…

Velocity head cont…

- α has values greater than unity depending on the degree of velocity variation across a section
- For laminar flow in a pipe, velocity distribution is parabolic across the section of the pipe, and α has value of 2.0
- However, if the flow is turbulent, as is the usual case for water flow through the large conduits, the velocity is fairly uniform over most of the conduit section, and α has value near unity (typically: $1.04 < \alpha < 1.06$).
- Therefore, in hydraulic engineering for ease of application in pipe flow, the value of α is usually assumed to be unity, and the velocity head is then simply $V^2/2g$.

Energy equation cont…

Pump or turbine head

- The head supplied by a pump is directly related to the power supplied to the flow as given below

$$P = Q\gamma h_p$$

- Likewise if head is supplied to turbine, the power supplied to the turbine will be

$$P = Q\gamma h_t$$

- These two equations represents the power supplied directly or power taken out directly from the flow

Energy equation cont…

Head-loss term

- The head loss term h_L accounts for the conversion of mechanical energy to internal energy (heat), when this conversion occurs, the internal energy is not readily converted back to useful mechanical energy, therefore it is called *head loss*

- Head loss results from viscous resistance to flow (friction) at the conduit wall or from the viscous dissipation of turbulence usually occurring with separated flow, such as in bends, fittings or outlet works.

Head loss calculation

- Head loss is due to friction between the fluid and the pipe wall and turbulence within the fluid
- The rate of head loss depend on roughness element size apart from velocity and pipe diameter
- Further the head loss also depends on whether the pipe is hydraulically smooth, rough or somewhere in between
- In water distribution system , head loss is also due to bends, valves and changes in pipe diameter

Head loss calculation

- Head loss for steady flow through a straight pipe:

$$\tau_0 A_w = \Delta p A_r$$

$$\Delta p = 4L\tau_0 / D$$

$$\tau_0 = f\rho V^2 / 8$$

$$h = \frac{\Delta p}{\gamma} = f\frac{L}{D}\frac{V^2}{2g}$$

- This is known as Darcy-Weisbach equation

- *h/L=S*, is slope of the hydraulic and energy grade lines for a pipe of constant diameter

Head loss calculation

Head loss in laminar flow:

- Hagen-Poiseuille equation gives $S = \frac{32V\mu}{D^2 \rho g}$

- Combining above with Darcy-Weisbach equation, gives f

$$f = \frac{64\mu}{\rho VD}$$

- Also we can write in terms of Reynolds number

$$f = \frac{64}{N_r}$$

- This relation is valid for N_r<1000

Head loss calculation

Head loss in turbulent flow:

- In turbulent flow, the friction factor is a function of both Reynolds number and pipe roughness
- As the roughness size or the velocity increases, flow is wholly rough and f depends on the relative roughness
- Where graphical determination of the friction factor is acceptable, it is possible to use a Moody diagram.
- This diagram gives the friction factor over a wide range of Reynolds numbers for laminar flow and smooth, transition, and rough turbulent flow

Head loss calculation

- The quantities shown in Moody Diagram are dimensionless so they can be used with any system of units
- Moody's diagram can be followed from any reference book

MINOR LOSSES

- Energy losses caused by valves, bends and changes in pipe diameter
- This is smaller than friction losses in straight sections of pipe and for all practical purposes ignored
- Minor losses are significant in valves and fittings, which creates turbulence in excess of that produced in a straight pipe

Head loss calculation

Minor losses can be expressed in three ways:

1. A minor loss coefficient K may be used to give head loss as a function of velocity head,

$$h = K\frac{V^2}{2g}$$

2. Minor losses may be expressed in terms of the equivalent length of straight pipe, or as pipe diameters (L/D) which produces the same head loss.

$$h = f\frac{L}{D}\frac{V^2}{2g}$$

Head loss calculation

1. A flow coefficient C_v which gives a flow that will pass through the valve at a pressure drop of 1psi may be specified. Given the flow coefficient the head loss can be calculated as

$$h = \frac{18.5 \times 10^6 D^4 V^2}{C_v^2 2g}$$

The flow coefficient can be related to the minor loss coefficient by

$$K = \frac{18.5 \times 10^6 D^2}{C_v^2}$$

Energy Equation for Flow in pipes

- Energy equation for pipe flow

$$z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_L$$

- The energy equation represents elevation, pressure, and velocity forms of energy. The energy equation for a fluid moving in a closed conduit is written between two locations at a distance (length) L apart. Energy losses for flow through ducts and pipes consist of major losses and minor losses.
- Minor Loss Calculations for Fluid Flow

$$h_m = K \frac{V^2}{2g}$$

- Minor losses are due to fittings such as valves and elbows

Major Loss Calculation for Fluid Flow

- Using Darcy-Weisbach Friction Loss Equation

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad \text{and} \quad V = \frac{Q}{A} \qquad \text{If non-circular duct, D computed from } D = \frac{4A}{P}$$

- Major losses are due to friction between the moving fluid and the inside walls of the duct.
- The Darcy-Weisbach method is generally considered more accurate than the Hazen-Williams method. Additionally, the Darcy-Weisbach method is valid for any liquid or gas.
- Moody Friction Factor Calculator

$$f = \frac{64}{Re} \quad \text{for } Re \le 2100 \text{ (laminar flow)} \qquad Re = \frac{VD}{\nu}$$

$$f = \frac{1.325}{\left[\ln\left(\frac{e}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \quad \text{for } 5000 \le Re \le 10^8 \text{ (turbulent flow) and } 10^{-6} \le \frac{e}{D} \le 10^{-2}$$

Major Loss Calculation in pipes

- Using Hazen-Williams Friction Loss Equation

$$V = k\,C\,R_h^{0.63}\,S^{0.54} \quad \text{where} \quad S = \frac{h_f}{L} \quad \& \quad Q = VA \quad \& \quad R_h = \frac{D}{4} \text{ for circular pipe}$$

- Hazen-Williams is only valid for water at ordinary temperatures (40 to 75^{o}F). The Hazen-Williams method is very popular, especially among civil engineers, since its friction coefficient (C) is not a function of velocity or duct (pipe) diameter. Hazen-Williams is simpler than Darcy-Weisbach for calculations where one can solve for flow-rate, velocity, or diameter

Transient flow through long pipes

- Intermediate flow while changing from one steady state to another is called transient flow
- This occurs due to design or operating errors or equipment malfunction.
- This transient state pressure causes lots of damage to the network system
- Pressure rise in a close conduit caused by an instantaneous change in flow velocity

Transient flow through long pipes

- If the flow velocity at a point does vary with time, the flow is unsteady
- When the flow conditions are changed from one steady state to another, the intermediate stage flow is referred to as transient flow
- The terms fluid transients and hydraulic transients are used in practice
- The different flow conditions in a piping system are discussed as below:

Transient flow through long pipes

- Consider a pipe length of length L
- Water is flowing from a constant level upstream reservoir to a valve at downstream
- Assume valve is instantaneously closed at time $t=t_0$ from the full open position to half open position.
- This reduces the flow velocity through the valve, thereby increasing the pressure at the valve

Transient flow through long pipes

- The increased pressure will produce a pressure wave that will travel back and forth in the pipeline until it is dissipated because of friction and flow conditions have become steady again

- This time when the flow conditions have become steady again, let us call it t_1.

- So the flow regimes can be categorized into

1. Steady flow for $t<t_0$
2. Transient flow for $t_0<t<t_1$
3. Steady flow for $t>t_1$

Transient flow through long pipes

- Transient-state pressures are sometimes reduced to the vapor pressure of a liquid that results in separating the liquid column at that section; this is referred to as liquid-column separation

- If the flow conditions are repeated after a fixed time interval, the flow is called periodic flow, and the time interval at which the conditions are repeated is called period

- The analysis of transient state conditions in closed conduits may be classified into two categories: lumped-system approach and distributed system approach

Transient flow through long pipes

- In the *lumped system* approach the conduit walls are assumed rigid and the liquid in the conduit is assumed incompressible, so that it behaves like a rigid mass, other way flow variables are functions of time only.

- In the *distributed system* approach the liquid is assumed slightly compressible

- Therefore flow velocity vary along the length of the conduit in addition to the variation in time

Transient flow through long pipes

Flow establishment

- The 1D form of momentum equation for a control volume that is fixed in space and does not change shape may be written as

$$\sum F = \frac{d}{dt}\int \rho V A dx + (\rho A V^2)_{out} - (\rho A V^2)_{in}$$

- If the liquid is assumed incompressible and the pipe is rigid, then at any instant the velocity along the pipe will be same,

$$(\rho A V^2)_{in} = (\rho A V^2)_{out}$$

Transient flow through long pipes

- Substituting for all the forces acting on the control volume

$$pA + \gamma AL \sin\alpha - \tau_0 \pi DL = \frac{d}{dt}(V\rho AL)$$

Where

$p = \gamma(h - V^2/2g)$

α=pipe slope

D=pipe diameter

L=pipe length

γ =specific weight of fluid

τ_0=shear stress at the pipe wall

Transient flow through long pipes

- Frictional force is replaced by $\gamma h_f A$, *and* $H_0 = h + L\sin\alpha$ and h_f from Darcy-weisbach friction equation
- The resulting equation yields:

$$H_0 - \frac{fL}{D}\frac{V^2}{2g} - \frac{V^2}{2g} = \frac{L}{g}\cdot\frac{dV}{dt}$$

- When the flow is fully established, $dV/dt=0$.
- The final velocity V_0 will be such that

$$H_0 = \left[1 + \frac{fL}{D}\right]\frac{V_0^2}{2g}$$

- We use the above relationship to get the time for flow to establish

$$dt = \frac{2LD}{D + fL}\cdot\frac{dV}{V_0^2 - V^2}$$

Transient flow through long pipes

Change in pressure due to rapid flow changes

- When the flow changes are rapid, the fluid compressibility is needed to taken into account
- Changes are not instantaneous throughout the system, rather pressure waves move back and forth in the piping system.
- Pipe walls to be rigid and the liquid to be slightly compressible

Transient flows through long pipes

- Assume that the flow velocity at the downstream end is changed from V to $V+\Delta V$, thereby changing the pressure from p to $p+\Delta p$

- The change in pressure will produce a pressure wave that will propagate in the upstream direction

- The speed of the wave be a

- The unsteady flow situation can be transformed into steady flow by assuming the velocity reference system move with the pressure wave

Transient flows through long pipes

- Using momentum equation with control volume approach to solve for Δp
- The system is now steady, the momentum equation now yield

$$pA-(p+\Delta p)A=(V+a+\Delta V)(\rho+\Delta\rho)(V+a+\Delta V)A-(V+a)\rho(V+a)A$$

- By simplifying and discarding terms of higher order, this equation becomes

$$-\Delta p=2\rho V\Delta V+2\rho\Delta Va+\Delta\rho\left(V^2+2Va+a^2\right)$$

- The general form of the equation for conservation of mass for one-dimensional flows may be written as

$$0=\frac{d}{dt}\int_{x_1}^{x_2}\rho A\,dx+\left(\rho VA\right)_{out}-\left(\rho VA\right)_{in}$$

Transient flows through long pipes

- For a steady flow first term on the right hand side is zero, then we obtain

$$0 = (\rho + \Delta\rho)(V + a + \Delta V)A - \rho(V + a)A$$

- Simplifying this equation, We have

$$\Delta\rho = -\frac{\rho \Delta V}{V + a}$$

- We may approximate *(V+a)* as a, because V<<a

$$\Delta\rho = -\frac{\rho \Delta V}{a}$$

- Since $\Delta p = \rho g \Delta H$ we can write as

$$\Delta H = -\frac{a}{g}\Delta V$$

- Note: change in pressure head due to an instantaneous change in flow velocity is approximately 100 times the change in the flow velocity