Computational Hydraulics

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Introduction to Numerical Analysis and Its Role in Computational Hydraulics

Module 3

2 lectures

Contents

- ***Numerical computing***
- ***Computer arithmetic***
- ***Parallel processing***
- ***Examples of problems needing numerical treatment***

What is computational hydraulics?

- It is one of the many fields of science in which the application of computers gives rise to a new way of working, which is intermediate between purely theoretical and experimental.
- The hydraulics that is reformulated to suit digital machine processes, is called computational hydraulics
- It is concerned with simulation of the flow of water, together with its consequences, using numerical methods on computers

What is computational hydraulics?

- There is not a great deal of difference with computational hydrodynamics or computational fluid dynamics, but these terms are too much restricted to the fluid as such.

- It seems to be typical of practical problems in hydraulics that they are rarely directed to the flow by itself, but rather to some consequences of it, such as forces on obstacles, transport of heat, sedimentation of a channel or decay of a pollutant.

Why numerical computing

- The higher mathematics can be treated by this method
- When there is no analytical solution, numerical analysis can deal such physical problems
- Example: *y = sin (x),* has no closed form solution.
- The following integral gives the length of one arch of the above curve

$$\int_0^{\pi} \sqrt{1+\cos^2(x)}\,dx$$

- Numerical analysis can compute the length of this curve by standard methods that apply to essentially any integrand
- Numerical computing helps in finding effective and efficient approximations of functions

Why Numerical computing?

- linearization of non linear equations
- Solves for a large system of linear equations
- Deals the ordinary differential equations of any order and complexity
- Numerical solution of Partial differential equations are of great importance in solving physical world problems
- Solution of initial and boundary value problems and estimates the eigen values and eigenvectors.
- Fit curves to data by a variety of methods

Computer arithmetic

- Numerical method is tedious and repetitive arithmetic, which is not possible to solve without the help of computer.
- On the other hand Numerical analysis is an approximation, which leads towards some degree of errors
- The errors caused by Numerical treatment are defined in terms of following:

- ***Truncation error*** : the e^x can be approximated through cubic polynomial as shown below

$$p_3(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$$

- e^x is an infinitely long series as given below and the error is due to the truncation of the series

$$e^x = p_3(x) + \sum_{n=4}^{\infty} \frac{x^n}{n!}$$

Computer arithmetic

- ***Round-off error*** : digital computers always use floating point numbers of fixed word length; the true values are not expressed exactly by such representations. Such error due to this computer imperfection is round-off error.

- ***Error in original data*** : any physical problem is represented through mathematical expressions which have some coefficients that are imperfectly known.

- ***Blunders*** : computing machines make mistakes very infrequently, but since humans are involved in programming, operation, input preparation, and output interpretation, blunders or gross errors do occur more frequently than we like to admit.

- ***Propagated error*** : propagated error is the error caused in the succeeding steps due to the occurrence of error in the earlier step, such error is in addition to the local errors. If the errors magnified continuously as the method continues, eventually they will overshadow the true value, destroying its validity, we call such a method *unstable*. For *stable* method (which is desired)– errors made at early points die out as the method continues.

Parallel processing

- It is a computing method that can only be performed on systems containing two or more processors operating simultaneously. Parallel processing uses several processors, all working on different aspects of the same program at the same time, in order to share the computational load

- For extremely large scale problems (short term weather forecasting, simulation to predict aerodynamics performance, image processing, artificial intelligence, multiphase flow in ground water regime etc), this speeds up the computation adequately.

Parallel processing

- Most computers have just one CPU, but some models have several. There are even computers with thousands of CPUs. With single-CPU computers, it is possible to perform parallel processing by connecting the computers in a network. However, this type of parallel processing requires very sophisticated software called distributed processing software.
- Note that parallel processing differs from multitasking, in which a single CPU executes several programs at once.

Parallel processing

Types of parallel processing job: In general there are three types of parallel computing jobs

- ***Parallel task***
- ***Parametric sweep***
- ***Task flow***

Parallel task

A parallel task can take a number of forms, depending on the application and the software that supports it. For a Message Passing Interface (MPI) application, a parallel task usually consists of a single executable running concurrently on multiple processors, with communication between the processes.

Parallel processing

Parametric Sweep

A parametric sweep consists of multiple instances of the same program, usually serial, running concurrently, with input supplied by an input file and output directed to an output file. There is no communication or interdependency among the tasks. Typically, the parallelization is performed exclusively (or almost exclusively) by the scheduler, based on the fact that all the tasks are in the same job.

Task flow

- A task flow job is one in which a set of unlike tasks are executed in a prescribed order, usually because one task depends on the result of another task.

Introduction to numerical analysis

- Any physical problem in hydraulics is represented through a set of differential equations.

- These equations describe the very fundamental laws of conservation of mass and momentum in terms of the partial derivatives of dependent variables.

- For any practical purpose we need to know the values of these variables instead of the values of their derivatives.

Introduction to numerical analysis

- These variables are obtained from integrating those ODEs/PDEs.
- Because of the presence of nonlinear terms a closed form solution of these equations is not obtainable, except for some very simplified cases
- Therefore they need to be analyzed numerically, for which several numerical methods are available
- Generally the PDEs we deal in the computational hydraulics is categorized as elliptic, parabolic and hyperbolic equations

Introduction to numerical analysis

The following methods have been used for numerical integration of the ODEs

- Euler method
- Modified Euler method
- Runge-Kutta method
- Predictor-Corrector method

Introduction to numerical analysis

The following methods have been used for numerical integration of the PDEs

- Characteristics method
- Finite difference method
- Finite element method
- Finite volume method
- Spectral method
- Boundary element method

Problems needing numerical treatment

- Computation of normal depth
- Computation of water-surface profiles
- Contaminant transport in streams through an advection-dispersion process
- Steady state Ground water flow system
- Unsteady state ground water flow system
- Flows in pipe network
- Computation of kinematic and dynamic wave equations