Computational Hydraulics

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Introduction to Hydraulics of Open Channels

Module 1

3 lectures

Topics to be covered

- ***Basic Concepts***
- ***Conservation Laws***
- ***Critical Flows***
- ***Uniform Flows***
- ***Gradually Varied Flows***
- ***Rapidly Varied Flows***
- ***Unsteady Flows***

Basic Concepts

- Open Channel flows deal with flow of water in open channels
- Pressure is atmospheric at the water surface and the pressure is equal to the depth of water at any section
- Pressure head is the ratio of pressure and the specific weight of water
- Elevation head or the datum head is the height of the section under consideration above a datum
- Velocity head ($=v^2/2g$) is due to the average velocity of flow in that vertical section

Basic Concepts Cont…

Total head = $p/\gamma + v^2/2g + z$

- Pressure head = p/γ
- Velocity head = $v^2/2g$
- Datum head = z

- The flow of water in an open channel is mainly due to head gradient and gravity
- Open Channels are mainly used to transport water for irrigation, industry and domestic water supply

Conservation Laws

The main conservation laws used in open channels are

Conservation Laws

- **Conservation of Mass**
- **Conservation of Momentum**
- **Conservation of Energy**

Conservation of Mass

Conservation of Mass

In any control volume consisting of the fluid (water) under consideration, the net change of mass in the control volume due to inflow and out flow is equal to the the net rate of change of mass in the control volume

- This leads to the classical continuity equation balancing the inflow, out flow and the storage change in the control volume.

- Since we are considering only water which is treated as incompressible, the density effect can be ignored

Conservation of Momentum and energy

Conservation of Momentum

This law states that the rate of change of momentum in the control volume is equal to the net forces acting on the control volume

- Since the water under consideration is moving, it is acted upon by external forces

- Essentially this leads to the Newton's second law

Conservation of Energy

This law states that neither the energy can be created or destroyed. It only changes its form.

Conservation of Energy

- Mainly in open channels the energy will be in the form of potential energy and kinetic energy
- Potential energy is due to the elevation of the water parcel while the kinetic energy is due to its movement
- In the context of open channel flow the total energy due these factors between any two sections is conserved
- This conservation of energy principle leads to the classical Bernoulli's equation

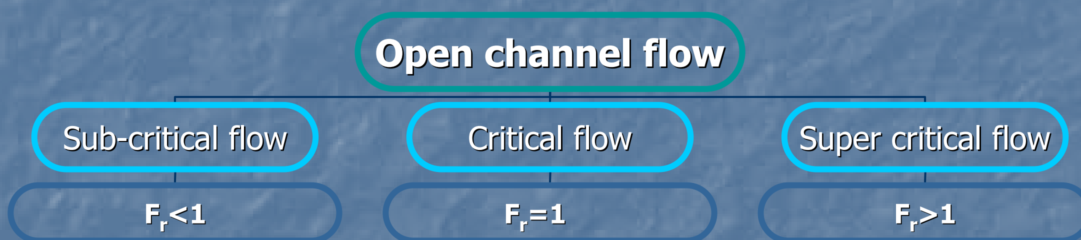
$P/\gamma + v^2/2g + z = Constant$

- When used between two sections this equation has to account for the energy loss between the two sections which is due to the resistance to the flow by the bed shear etc.

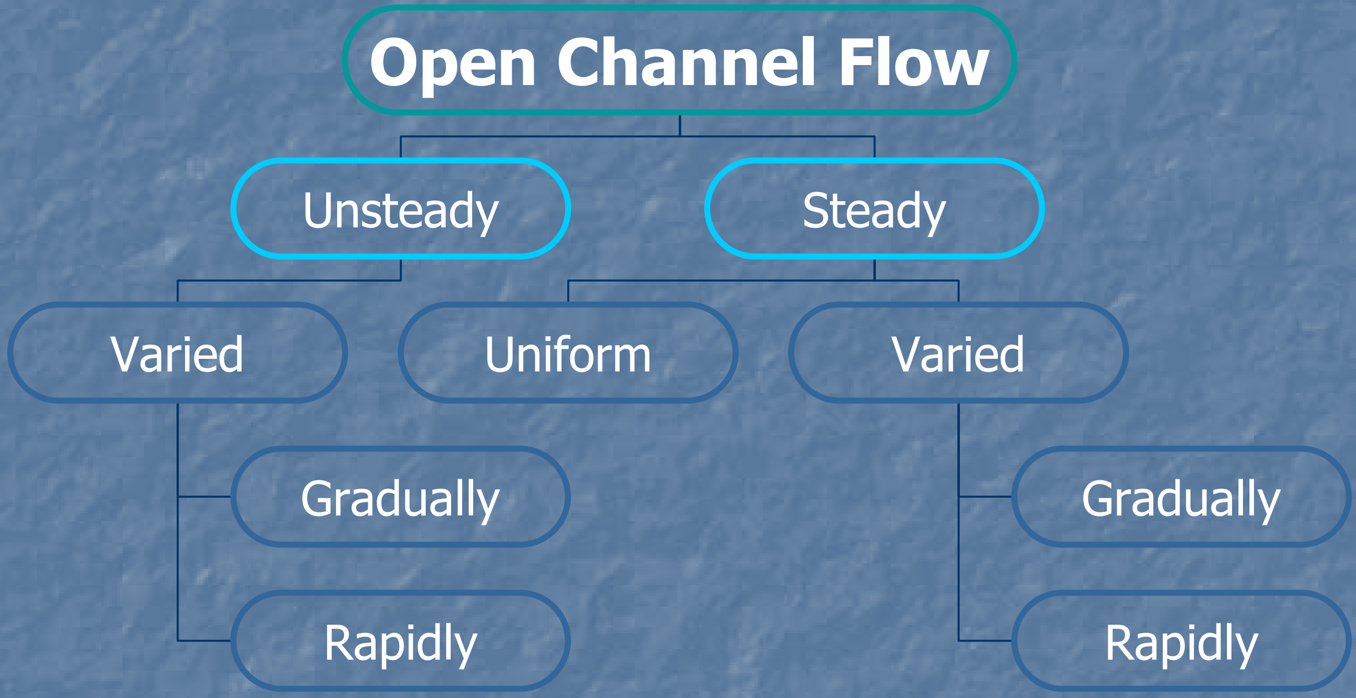
Types of Open Channel Flows

Depending on the Froude number (F_r) the flow in an open channel is classified as ***Sub critical*** *flow,* ***Super Critical*** *flow, and* ***Critical*** *flow, where Froude number can be defined as*

$$F_r = \frac{V}{\sqrt{gy}}$$



Types of Open Channel Flow Cont...



Types of Open Channel Flow Cont…

- ***Steady Flow***

Flow is said to be steady when discharge does not change along the course of the channel flow

- ***Unsteady Flow***

Flow is said to be unsteady when the discharge changes with time

- ***Uniform Flow***

Flow is said to be uniform when both the depth and discharge is same at any two sections of the channel

Types of Open Channel Cont…

- ***Gradually Varied Flow***

Flow is said to be gradually varied when ever the depth changes gradually along the channel

- ***Rapidly varied flow***

Whenever the flow depth changes rapidly along the channel the flow is termed rapidly varied flow

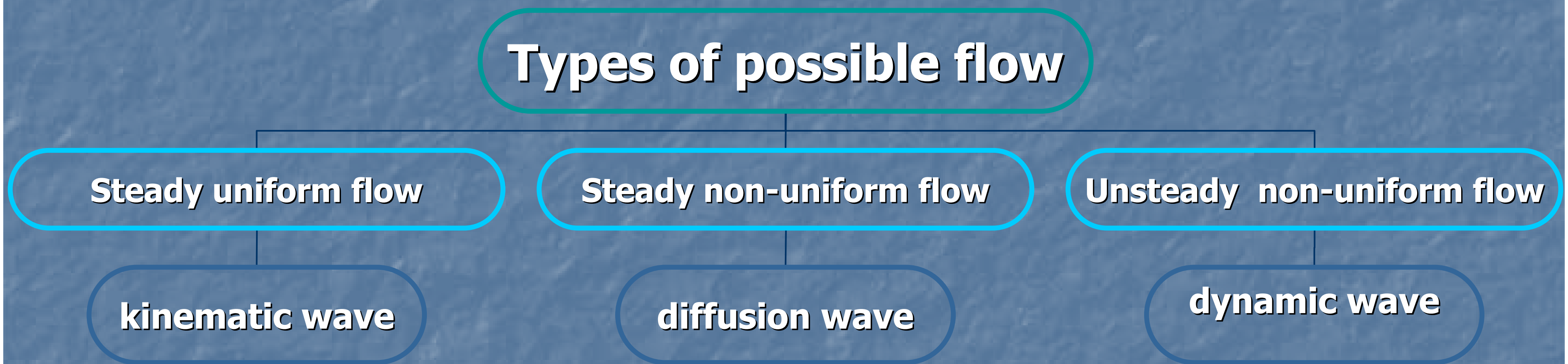
- ***Spatially varied flow***

Whenever the depth of flow changes gradually due to change in discharge the flow is termed spatially varied flow

Types of Open Channel Flow cont…

- ***Unsteady Flow***

Whenever the discharge and depth of flow changes with time, the flow is termed unsteady flow



Definitions

Specific Energy

It is defined as the energy acquired by the water at a section due to its depth and the velocity with which it is flowing

- Specific Energy E is given by, $E = y + v^2/2g$

Where y is the depth of flow at that section and v is the average velocity of flow

- Specific energy is minimum at critical condition

Definitions

Specific Force

It is defined as the sum of the momentum of the flow passing through the channel section per unit time per unit weight of water and the force per unit weight of water

$$F = Q^2/gA + yA$$

- The specific forces of two sections are equal provided that the external forces and the weight effect of water in the reach between the two sections can be ignored.

- At the critical state of flow the specific force is a minimum for the given discharge.

Critical Flow

Flow is critical when the specific energy is minimum. Also whenever the flow changes from sub critical to super critical or vice versa the flow has to go through critical condition

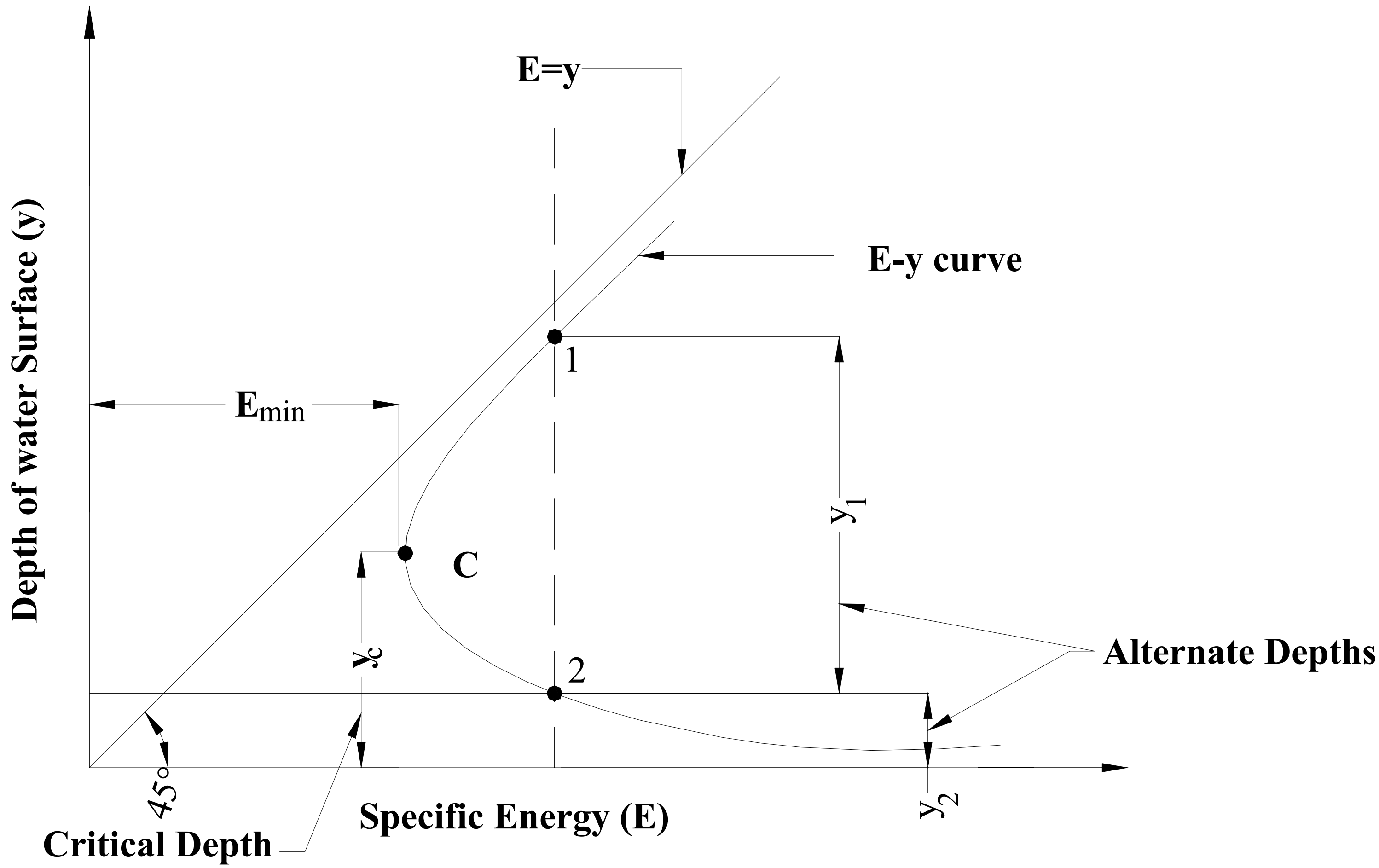
figure is shown in next slide

- **Sub-critical** flow-the depth of flow will be higher whereas the velocity will be lower.

- **Super-critical** flow-the depth of flow will be lower but the velocity will be higher

- **Critical flow**: Flow over a free over-fall

Specific energy diagram



Specific Energy Curve for a given discharge

Characteristics of Critical Flow

- Specific Energy (*$E = y+Q^2/2gA^2$*) is minimum

- For Specific energy to be a minimum *$dE/dy = 0$*

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3} \cdot \frac{dA}{dy}$$

- *However, $dA=Tdy$,* where T is the width of the channel at the water surface, then applying *$dE/dy = 0$, will result in following*

$$\frac{Q^2 T_c}{g A_c^{\,3}} = 1 \qquad \frac{A_c}{T_c} = \frac{Q^2}{g A_c^{\,2}} \qquad \frac{A_c}{T_c} = \frac{V_C^2}{g}$$

Characteristics of Critical Flow

- For a rectangular channel $A_c/T_c=y_c$
- Following the derivation for a rectangular channel,

$$F_r = \frac{V_c}{\sqrt{gy_c}} = 1$$

- The same principle is valid for trapezoidal and other cross sections
- Critical flow condition defines an unique relationship between depth and discharge which is very useful in the design of flow measurement structures

Uniform Flows

- This is one of the most important concept in open channel flows

- The most important equation for uniform flow is Manning's equation given by

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

Where R = the hydraulic radius = A/P

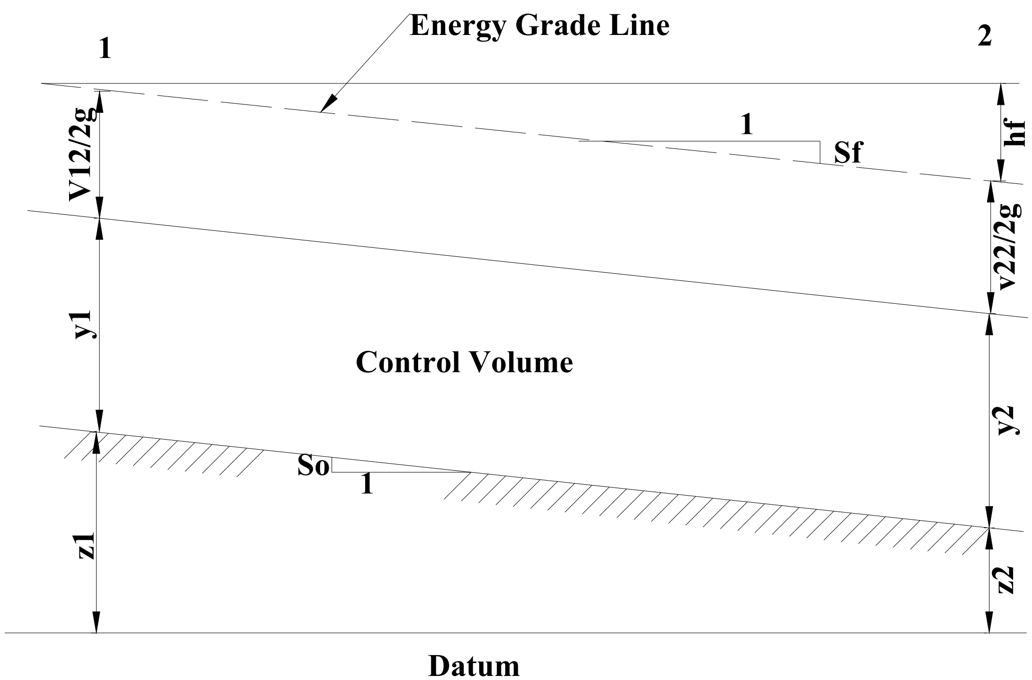
P = wetted perimeter = $f(y, S_0)$

Y = depth of the channel bed

S_0 = bed slope (same as the energy slope, S_f)

n = the Manning's dimensional empirical constant

Uniform Flows



Steady Uniform Flow in an Open Channel

Uniform Flow

- Example : Flow in an open channel
- This concept is used in most of the open channel flow design
- The uniform flow means that there is no acceleration to the flow leading to the weight component of the flow being balanced by the resistance offered by the bed shear
- In terms of discharge the Manning's equation is given by

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

Uniform Flow

- This is a non linear equation in y the depth of flow for which most of the computations will be made

- Derivation of uniform flow equation is given below, where

$W \sin\theta$ = weight component of the fluid mass in the direction of flow

τ_0 = bed shear stress

$P\Delta x$ = surface area of the channel

Uniform Flow

- The force balance equation can be written as

$$W \sin\theta - \tau_0 P \Delta x = 0$$

- Or

$$\gamma A \Delta x \sin\theta - \tau_0 P \Delta x = 0$$

- Or

$$\tau_0 = \gamma \frac{A}{P} \sin\theta$$

- Now *A/P* is the hydraulic radius, *R*, and *sinθ* is the slope of the channel S_0

Uniform Flow

- The shear stress can be expressed as

$$\tau_0 = c_f \rho \left(V^2 / 2 \right)$$

- Where c_f is resistance coefficient, V is the mean velocity ρ is the mass density
- Therefore the previous equation can be written as
- Or

$$c_f \rho \frac{V^2}{2} = \gamma R S_0$$

$$V = \sqrt{\frac{2g}{c_f}} \sqrt{R S_0} = C \sqrt{R S_0}$$

- where C is Chezy's constant
- For Manning's equation

$$C = \frac{1.49}{n} R^{1/6}$$

Gradually Varied Flow

Flow is said to be gradually varied whenever the depth of flow changed gradually

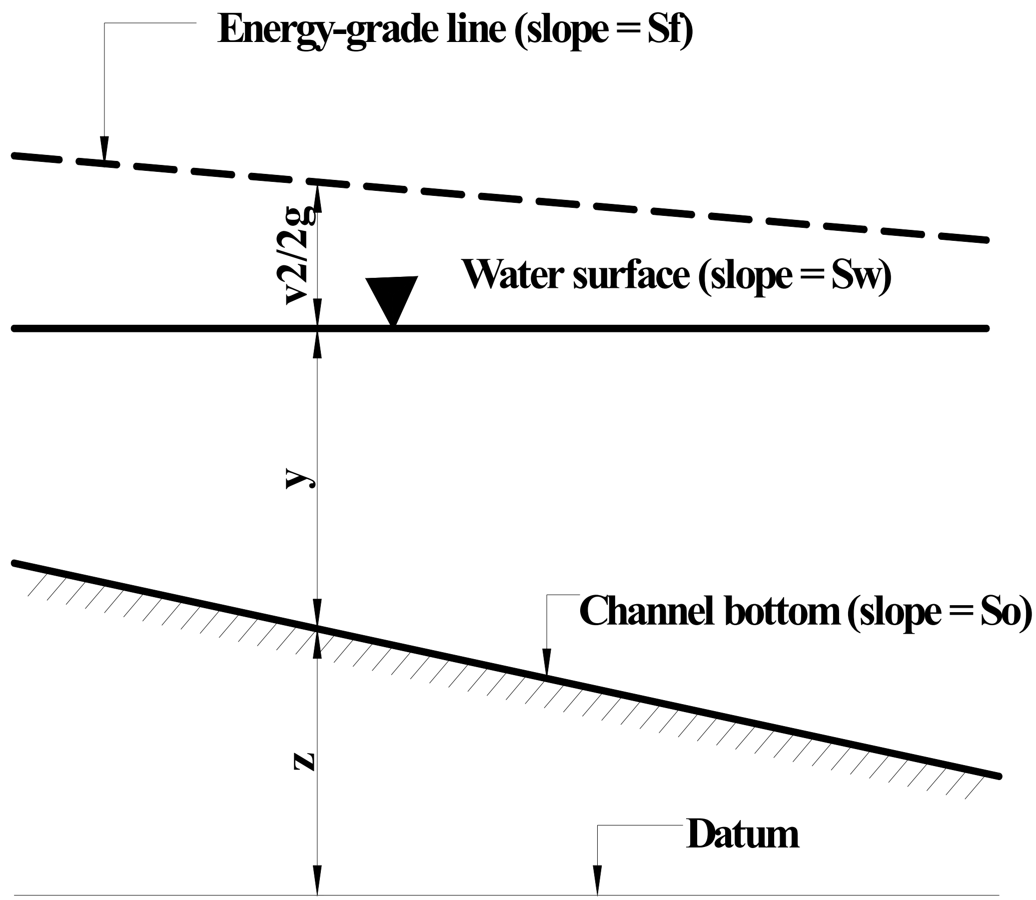
- The governing equation for gradually varied flow is given by

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^{\ 2}}$$

- Where the variation of depth *y* with the channel distance *x* is shown to be a function of bed slope S_0, Friction Slope S_f and the flow Froude number F_r.

- This is a non linear equation with the depth varying as a non linear function

Gradually Varied Flow



Total head at a channel section

Gradually Varied Flow

Derivation of gradually varied flow is as follows…

- The conservation of energy at two sections of a reach of length *Δx*, can be written as

$$y_1 + \frac{V_1^{\ 2}}{2g} + S_0 \Delta x = y_2 + \frac{V_2^{\ 2}}{2g} + S_f \Delta x$$

- Now, let $\Delta y = y_2 - y_1$ and $\frac{V_2^{\ 2}}{2g} - \frac{V_1^{\ 2}}{2g} = \frac{d}{dx}\left(\frac{V^2}{2g}\right)\Delta x$

Then the above equation becomes

$$\Delta y = S_0 \Delta x - S_f \Delta x - \frac{d}{dx}\left(\frac{V^2}{2g}\right)\Delta x$$

Gradually Varied Flow

- Dividing through *Δx* and taking the limit as *Δx* approaches zero gives us

$$\frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) = S_0 - S_f$$

- After simplification,

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 + d\left(V^2/2g\right)/dy}$$

- Further simplification can be done in terms of Froude number

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right)$$

Gradually Varied Flow

- After differentiating the right side of the previous equation,

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{-2Q^2}{2gA^3} \cdot \frac{dA}{dy}$$

- But *dA/dy=T, and A/T=D,* therefore,

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{-Q^2}{gA^2 D} = -F_r^{\,2}$$

- Finally the general differential equation can be written as

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^{\,2}}$$

Gradually Varied Flow

- Numerical integration of the gradually varied flow equation will give the water surface profile along the channel

- Depending on the depth of flow where it lies when compared with the normal depth and the critical depth along with the bed slope compared with the friction slope different types of profiles are formed such as M (mild), C (critical), S (steep) profiles. All these have real examples.

- M (mild)-If the slope is so small that the normal depth (Uniform flow depth) is greater than critical depth for the given discharge, then the slope of the channel is **mild**.

Gradually Varied Flow

- C (critical)-if the slope's normal depth equals its critical depth, then we call it a **critical** slope, denoted by C
- S (steep)-if the channel slope is so steep that a normal depth less than critical is produced, then the channel is **steep**, and water surface profile designated as S

Rapidly Varied Flow

- This flow has very pronounced curvature of the streamlines
- It is such that pressure distribution cannot be assumed to be hydrostatic
- The rapid variation in flow regime often take place in short span
- When rapidly varied flow occurs in a sudden-transition structure, the physical characteristics of the flow are basically fixed by the boundary geometry of the structure as well as by the state of the flow

Examples:

- Channel expansion and cannel contraction
- Sharp crested weirs
- Broad crested weirs

Unsteady flows

- When the flow conditions vary with respect to time, we call it unsteady flows.

- Some terminologies used for the analysis of unsteady flows are defined below:

- ***Wave:*** it is defined as a temporal or spatial variation of flow depth and rate of discharge.

- ***Wave length***: it is the distance between two adjacent wave crests or trough

- ***Amplitude***: it is the height between the maximum water level and the still water level

Unsteady flows definitions

- ***Wave celerity** (c):* relative velocity of a wave with respect to fluid in which it is flowing with *V*
- ***Absolute wave velocity** (V_w):* velocity with respect to fixed reference as given below

$$V_w = V \pm c$$

- Plus sign if the wave is traveling in the flow direction and minus for if the wave is traveling in the direction opposite to flow
- For shallow water waves $c = \sqrt{g y_0}$ where y_0=undisturbed flow depth.

Unsteady flows examples

Unsteady flows occur due to following reasons:

1. Surges in power canals or tunnels
2. Surges in upstream or downstream channels produced by starting or stopping of pumps and opening and closing of control gates
3. Waves in navigation channels produced by the operation of navigation locks
4. Flood waves in streams, rivers, and drainage channels due to rainstorms and snowmelt
5. Tides in estuaries, bays and inlets

Unsteady flows

- Unsteady flow commonly encountered in an open channels and deals with translatory waves. Translatory waves is a gravity wave that propagates in an open channel and results in appreciable displacement of the water particles in a direction parallel to the flow

- For purpose of analytical discussion, unsteady flow is classified into two types, namely, gradually varied and rapidly varied unsteady flow

- In gradually varied flow the curvature of the wave profile is mild, and the change in depth is gradual

- In the rapidly varied flow the curvature of the wave profile is very large and so the surface of the profile may become virtually discontinuous.

Unsteady flows cont…

- Continuity equation for unsteady flow in an open channel

$$D\frac{\partial V}{\partial x}+V\frac{\partial y}{\partial x}+\frac{\partial y}{\partial t}=0$$

- For a rectangular channel of infinite width, may be written

$$\frac{\partial q}{\partial x}+\frac{\partial y}{\partial t}=0$$

- When the channel is to feed laterally with a supplementary discharge of q′ per unit length, for instance, into an area that is being flooded over a dike

Unsteady flows cont…

- The equation

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}+q^{'}=0$$

- The general dynamic equation for gradually varied unsteady flow is given by:

$$\frac{\partial y}{\partial x}+\frac{\alpha V}{g}\frac{\partial V}{\partial x}+\frac{1}{g}\frac{\partial V}{\partial t}^{'}=0$$