

Computational Hydraulics



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Computation of Gradually Varied and Unsteady Open Channel Flows

Module 9

6 lectures

Contents

- *Numerical integration methods for solving Gradually varied flows*
- *Finite difference methods for Saint Venant-equations*
- *Examples*



Introduction

- For most of the practical implications, the flow conditions in a gradually varied flow are required to calculate.
- These calculations are performed to determine the water surface elevations required for the planning, design, and operation of open channels so that the effects of the addition of engineering works and the channel modifications on water levels may be assessed
- Also steady state flow conditions are needed to specify proper initial conditions for the computation of unsteady flows

Introduction

- Improper initial conditions introduce false transients into the simulation, which may lead to incorrect results
- It is possible to use unsteady flow algorithms directly to determine the initial conditions by computing for long simulation time
- However, such a procedure is computationally inefficient and may not converge to the proper steady state solution if the finite-difference scheme is not consistent

Introduction

- Various methods to compute gradually varied flows are required to develop
- Methods, which are suitable for a computer solution, are adopted
- Traditionally there are two methods-direct and standard step methods
- Higher order accurate methods to numerically integrate the governing differential equation are required

Equation of gradually varied flow

- Consider the profile of gradually varied flow in the elementary length dx of an open channel.
- The total head above the datum at the upstream section is

$$H = z + d \cos \theta + \alpha \frac{V^2}{2g}$$

H = total head

z = vertical distance of the channel bottom above the datum

d = depth of flow section

θ = bottom slope angle

α = energy coefficient

V = mean velocity of flow through the section

Equation of gradually varied flow

- Differentiating

$$\frac{dH}{dx} = \frac{dz}{dx} + \cos \theta \frac{dd}{dx} + \alpha \frac{d}{dx} \left(\frac{V^2}{2g} \right)$$

- The energy slope, $S_f = -dH / dx$
- The slope of the channel bottom, $S_0 = \sin \theta = -dz / dx$
- Substituting these slopes in above equations and solving for dd/dx ,

$$\frac{dd}{dx} = \frac{S_0 - S_f}{\cos \theta + \alpha d(V^2 / 2g) / dd}$$

Equation of gradually varied flow

- This is the general differential equation for gradually varied flow
- For small θ , $\cos\theta \approx 1$, $d \approx y$, and $dd/dx \approx dy/dx$, thus the above equation becomes,

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 + \alpha d(V^2 / 2g) / dy}$$

- Since $V = Q/A$, and $dA/dy = T$, the velocity head term may be expressed as

$$\alpha \frac{d}{dy} \left(\frac{V^2}{2g} \right) = \frac{\alpha Q^2}{2g} \frac{dA^{-2}}{dy} = -\frac{\alpha Q^2}{gA^3} \frac{dA}{dy} = -\frac{\alpha Q^2 T}{gA^3}$$

Equation of gradually varied flow

- Since, $Z = \sqrt{A^3 / T}$

- The above may be written as

$$\alpha \frac{d}{dy} \left(\frac{V^2}{2g} \right) = - \frac{\alpha Q^2}{gZ^2}$$

- Suppose that a critical flow of discharge equal to Q occurs at the section;

$$Q = Z_c \sqrt{\frac{g}{\alpha}}$$

- After substituting

$$\alpha \frac{d}{dy} \left(\frac{V^2}{2g} \right) = - \frac{Z_c^2}{Z^2}$$

Equation of gradually varied flow

- When the Manning's formula is used, the energy slope is

$$S_f = \frac{n^2 V^2}{2.22 R^{4/3}}$$

- When the Chezy formula is used,

$$S_f = \frac{V^2}{C^2 R}$$

- In general form,

$$S_f = \frac{Q^2}{K^2}$$

Computation of gradually varied flows

- The analysis of continuity, momentum, and energy equations describe the relationships among various flow variables, such as the flow depth, discharge, and flow velocity throughout a specified channel length
- The channel cross section, Manning n , bottom slope, and the rate of discharge are usually known for these steady-state-flow computations.
- The rate of change of flow depth in gradually varied flows is usually small, such that the assumption of hydrostatic pressure distribution is valid

Computation of gradually varied flows

The graphical-integration method:

- Used to integrate dynamic equation graphically
- Two channel sections are chosen at x_1 and x_2 with corresponding depths of flow y_1 and y_2 , then the distance along the channel floor is

$$x = x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{y_1}^{y_2} \frac{dx}{dy} dy$$

- Assuming several values of y , and computing the values of dx/dy
- A curve of y against dx/dy is constructed

Computation of gradually varied flows

- The value of x is equal to the shaded area formed by the curve, y -axis, and the ordinates of dx/dy corresponding to y_1 and y_2 .
- This area is measured and the value of x is determined.
- It applies to flow in prismatic as well as non-prismatic channels of any shape and slope
- This method is easier and straightforward to follow.

Computation of gradually varied flows

Method of direct integration

- Gradually varied flow cannot be expressed explicitly in terms of y for all types of channel cross section
- Few special cases has been solved by mathematical integration

Use of numerical integration for solving gradually varied flows

- Total head at a channel section may be written as

$$H = z + y + \frac{\alpha V^2}{2g}$$

Where

H = elevation of energy line above datum;

z = elevation of the channel bottom above the datum;

y = flow depth;

V = mean flow velocity, and

α = velocity-head coefficient

- The rate of variation of flow depth, y , with respect to distance x is obtained by differentiating the above equation.

Solution of gradually varied flows

- Consider x positive in the downstream flow direction
- By differentiating the above energy equation, we get the water surface profile as

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - (\alpha Q^2 B)/(gA^3)}$$

- The above equation is of first order ordinary differential equation, in which x is independent variable and y is the dependent variable.

Solution of gradually varied flows

In the above differential equation for gradually varied flows, the parameters are as given below:

x = distance along the channel (positive in downward direction)

S_0 = longitudinal slope of the channel bottom

S_f = slope of the energy line

B = top water surface width

g = acceleration due to gravity

A = flow area

Q = rate of discharge

Solution of gradually varied flows

- The right hand of the above equation shows that it is a function of x and y , so assume this function as $f(x, y)$, then we can write above equation as

$$\frac{dy}{dx} = f(x, y)$$

- In which,

$$f(x, y) = \frac{S_o - S_f}{1 - (\alpha Q^2 B)/(gA^3)}$$

- We can integrate above differential equation to determine the flow depth along a channel length, where $f(x, y)$ is nonlinear function. So the numerical methods are useful for its integration.

Solution of gradually varied flows

- These methods yields flow depth discretely
- To determine the value y^2 at distance x^2 , we proceed as follows

$$\int_{y_1}^{y_2} dy = \int_{x_1}^{x_2} f(x, y) dx$$

- The above integration yields..

$$y_2 = y_1 + \int_{x_1}^{x_2} f(x, y) dx$$

Solution of gradually varied flows

- We the y values along the downstream if dx is positive and upstream values if dx is negative
- We numerically evaluate the integral term
- Successive application provides the water surface profile in the desired channel length
- To determine x^2 where the flow depth will be y^2 , we proceed as follows:

$$\frac{dx}{dy} = F(x, y)$$

Solution of gradually varied flows

- In which

$$F(x, y) = \frac{1 - (\alpha Q^2 B) / (g A^3)}{S_o - S_f}$$

- Integrating the above differential equation we get,

$$x_2 = x_1 + \int_{y_1}^{y_2} F(x, y) dy$$

- To compute the water surface profile, we begin the computations at a location where the flow depth for the specified discharge is known
- We start the computation at the downstream control section if the flow is sub-critical and proceed in the upstream direction.

Solution of gradually varied flows

- In supercritical flows, however, we start at an upstream control section and compute the profile in the downstream direction
- This is due to the fact that the flow depth is known at only control section, we proceed in either the upstream or downstream direction.
-
- In the previous sections we discussed how to compute the locations where a specified depth will occur
- A systematic approach is needed to develop for these computations
- A procedure called *direct step method* is discussed below

Solution of gradually varied flows

Direct step method

- Assume the properties of the channel section are known then,

$$z_2 = z_1 - S_0(x_2 - x_1)$$

- In addition, the specific energy

$$E_1 = y_1 + \frac{\alpha_1 V_1^2}{2g}$$

$$E_2 = y_2 + \frac{\alpha_2 V_2^2}{2g}$$

- The slope of the energy grade line is gradually varied flow may be computed with negligible error by using the corresponding formulas for friction slopes in uniform flow.

Solution of gradually varied flows

- The following approximations have been used to select representative value of S_f for the channel length between section 1 and 2

- Average friction slope

$$\bar{S}_f = \frac{1}{2}(S_{f_1} + S_{f_2})$$

- Geometric mean friction slope

$$\bar{S}_f = \sqrt{S_{f_1} S_{f_2}}$$

- Harmonic mean friction slope

$$\bar{S}_f = \frac{2S_{f_1} S_{f_2}}{S_{f_1} + S_{f_2}}$$

Solution of gradually varied flows

- The friction loss may be written as

$$h_f = \frac{1}{2}(S_{f_1} + S_{f_2})(x_2 - x_1)$$

- From the energy equation we can write,

$$z_1 + E_1 = z_2 + E_2 + \frac{1}{2}(S_{f_1} + S_{f_2})(x_2 - x_1)$$

- Writing in terms of bed slope

$$x_2 = x_1 + \frac{E_2 - E_1}{S_o - \frac{1}{2}(S_{f_1} + S_{f_2})}$$

- Now from the above equation, the location of section 2 is known.

Solution of gradually varied flows

- This is now used as the starting value for the next step
- Then by successively increasing or decreasing the flow depth and determining where these depths will occur, the water surface profile in the desired channel length may be computed
- The direction of computations is automatically taken care of if proper sign is used for the numerator and denominator

Solution of gradually varied flows

The disadvantages of this method are

1. The flow depth is not computed at predetermined locations. Therefore, interpolations may become necessary, if the flow depths are required at specified locations. Similarly, the cross-sectional information has to be estimated if such information is available only at the given locations. This may not yield accurate results
2. Needs additional effort
3. It is cumbersome to apply to non-prismatic channels

Solution of gradually varied flows

Standard step method

- When we require to determine the depth at specified locations or when the channel cross sections are available only at some specified locations, the direct step method is not suitable enough to apply and in these cases standard step method is applied
- In this method the following steps are followed :
- Total head at section 1

$$H_1 = z_1 + y_1 + \frac{\alpha_1 V_1^2}{2g}$$

Solution of gradually varied flows

- Total head at section 2

$$H_2 = H_1 - h_f$$

- Including the expression for friction loss h_f

$$H_2 = H_1 - \frac{1}{2}(S_{f_1} + S_{f_2})(x_2 - x_1)$$

- Substituting the total head at 2 in terms of different heads, we obtain

$$y_2 + \frac{\alpha_2 Q^2}{2gA_2^2} + \frac{1}{2}S_{f_2}(x_2 - x_1) + z_2 - H_1 + \frac{1}{2}S_{f_1}(x_2 - x_1) = 0$$

Solution of gradually varied flows

- In the above equation. A_2 and S_{f_2} are functions of y_2 , and all other quantities are either known or already have been calculated at section 1.
- The flow depth y_2 is then determined by solving the following nonlinear algebraic equation:

$$F(y_2) = y_2 + \frac{\alpha_2 Q^2}{2gA_2^2} + \frac{1}{2} S_{f_2} (x_2 - x_1) + z_2 - H_1 + \frac{1}{2} S_{f_1} (x_2 - x_1) = 0$$

- The above equation is solved for y_2 by a trial and error procedure or by using the Newton or Bisection methods

Solution of gradually varied flows

- Here the Newton method is discussed.
- For this method we need an expression for dF/dy_2

$$\frac{dF}{dy_2} = 1 - \frac{\alpha_2 Q^2}{g A_2^3} \frac{dA_2}{dy_2} + \frac{1}{2} (x_2 - x_1) \frac{d}{dy_2} \left(\frac{Q^2 n^2}{C_o^2 A_2^2 R_2^{4/3}} \right)$$

- The last term of the above equations can be evaluated as

$$\begin{aligned} \frac{d}{dy_2} \left(\frac{Q^2 n^2}{C_o^2 A_2^2 R_2^{4/3}} \right) &= \frac{-2Q^2 n^2}{C_o^2 A_2^2 R_2^{4/3}} \frac{dA_2}{dy_2} - \frac{4}{3} \frac{Q^2 n^2}{C_o^2 A_2^2 R_2^{7/3}} \frac{dR_2}{dy_2} \\ &= \frac{-2Q^2 n^2}{C_o^2 A_2^2 R_2^{4/3}} \frac{dA_2}{dy_2} - \frac{4}{3} \frac{Q^2 n^2}{C_o^2 A_2^2 R_2^{4/3}} \frac{1}{R_2} \frac{dR_2}{dy_2} \\ &= -2 \left(S_{f_2} \frac{B_2}{A_2} + \frac{2}{3} \frac{S_{f_2}}{R_2} \frac{dR_2}{dy_2} \right) \end{aligned}$$

Solution of gradually varied flows

- Here dA_2/dy_2 is replaced by B_2 in the above equation and substituting for this expression

$$\frac{dF}{dy_2} = 1 - \frac{\alpha_2 Q^2 B_2}{g A_2^3} - (x_2 - x_1) \left(S_{f_2} \frac{B_2}{A_2} + \frac{2}{3} \frac{S_{f_2}}{R_2} \frac{dR_2}{dy_2} \right)$$

- By using $y=y_1$, $dy/dx=f(x_1, y_1)$, then the flow depth y_2^* , can be computed from the equation

$$y_2^* = y_1 + f(x_1, y_1)(x_2 - x_1)$$

- During subsequent step, however y_2^* may be determined by extrapolating the change in flow depth computed during the preceding step.

Solution of gradually varied flows

- A better estimate for y_2 can be computed from the equation

$$y_2 = y_2^* - \frac{F(y_2^*)}{[dF / dy_2]^*}$$

- If $|y_2 - y_2^*|$ is less than a specified tolerance, ε , then y_2^* is the flow depth y_2 , at section 2; otherwise, set $y_2^* = y_2$ and repeat the steps until a solution is obtained

Solution of gradually varied flows

Integration of differential equation

- For the computation of the water surface profile by integrating the differential equation, the integration has to be done numerically, since $f(x,y)$ is a nonlinear function
- Different numerical methods have been developed to solve such nonlinear system efficiently
- The numerical methods that are in use to evaluate the integral term can be divided into following categories:
 1. **Single-step methods**
 2. **Predictor-corrector methods**

Solution of gradually varied flows

- The single step method is similar to direct step method and standard step method
- The unknown depths are expressed in terms of a function $f(x, y)$, at a neighboring point where the flow depth is either initially known or calculated during the previous step
- In the predictor-corrector method the value of the unknown is first predicted from the previous step
- This predicted value is then refined through iterative process during the corrector part till the solution is reached by the convergence criteria

Solution of gradually varied flows

Single-step methods

Euler method

Modified Euler method

Improved Euler method

Fourth-order Runge-Kutta method

1. Euler method: In this method the rate of variation of y with respect to x at distance x_i can be estimated as

$$y'_i = \left. \frac{dy}{dx} \right|_i = f(x_i, y_i)$$

Solution of gradually varied flows

- The rate of change of depth of flow in a gradually varied flow is given as below

$$f(x_i, y_i) = \frac{S_o - S_{fi}}{1 - (\alpha Q^2 B_i) / (g A_i^3)}$$

- All the variables are known in the right hand side, so derivative of y with respect to x can be obtained
- Assuming that this variation is constant in the interval x_i to x_{i+1} , then the flow depth at x_{i+1} can be computed from the equation

$$y_{i+1} = y_i + f(x_i, y_i)(x_{i+1} - x_i)$$

Solution of gradually varied flows cont..

2. Modified Euler method

We may also improve the accuracy of the Euler method by using the slope of the curve $y = y(x)$ at $x = x_{i+1/2}$ and

$$y = y_{i+1/2}, \text{ in which } x_{i+1/2} = \frac{1}{2}(x_i + x_{i+1}) \text{ and } y_{i+1/2} = y_i + \frac{1}{2}y'_i\Delta x .$$

Let us call this slope $y'_{i+1/2}$. Then

$$y_{i+1} = y_i + y'_{i+1/2}\Delta x$$

or

$$y_{i+1} = y_i + f(x_{i+1/2}, y_{i+1/2})\Delta x$$

This method, called the modified Euler method, is second-order accurate.

Solution of gradually varied flows cont..

3. Improved Euler method

Let us call the flow depth at x_{i+1} obtained by using Euler method as y_{i+1}^* i.e.,
$$y_{i+1}^* = y_i + y_i' \Delta x$$

By using this value, we can compute the slope of the curve $y = y(x)$ at $x = x_{i+1}$, i.e., $y_{i+1}' = f(x_{i+1}, y_{i+1}^*)$ Let us

use the average value of the slopes of the curve at x_i and x_{i+1} . Then we can determine the value of y_{i+1} from the

equation $y_{i+1} = y_i + \frac{1}{2}(y_i' + y_{i+1}') \Delta x$. This equation may be

written as $y_{i+1} = y_i + \frac{1}{2}[f(x_i, y_i) + f(x_{i+1}, y_{i+1}^*)] \Delta x$. This method called the improved Euler method, is second order accurate.

Solution of gradually varied flows cont..

4. Fourth-order Runge Kutta Method

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{1}{2}\Delta x, y_i + \frac{1}{2}k_1\Delta x\right)$$

$$k_3 = f\left(x_i + \frac{1}{2}\Delta x, y_i + \frac{1}{2}k_2\Delta x\right)$$

$$k_4 = f(x_i + \Delta x, y_i + k_3\Delta x)$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\Delta x$$

Solution of gradually varied flows cont..

Predictor-corrector methods

- In this method we predict the unknown flow depth first, correct this predicted value, and then re-correct this corrected value. This iteration is continued till the desired accuracy is met.
- In the predictor part, let us use the Euler method to predict the value of y_{i+1} , I.e

$$y_{i+1}^{(0)} = y_i + f(x_i, y_i)\Delta x$$

- we may correct using the following equation

$$y_{i+1}^{(1)} = y_i + \frac{1}{2}[f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{(0)})]\Delta x$$

Solution of gradually varied flows cont..

- Now we may re-correct y again to obtain a better value:

$$y_{i+1}^{(2)} = y_i + \frac{1}{2}[f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{(1)})]\Delta x$$

- Thus the j th iteration is

$$y_{i+1}^{(j)} = y_i + \frac{1}{2}[f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{(j-1)})]\Delta x$$

- Iteration until tolerance $\left| y_{i+1}^{(j)} - y_{i+1}^{(j-1)} \right| \leq \varepsilon$, where $\varepsilon =$ specified tolerance

Saint-Venant equations

- 1D gradually varied unsteady flow in an open channel is given by Saint-Venant equations

$$a \frac{\partial v}{\partial x} + vw \frac{\partial y}{\partial x} + w \frac{\partial y}{\partial t} = 0$$

$$v \frac{\partial v}{\partial x} + g \frac{\partial y}{\partial x} + \frac{\partial v}{\partial t} = g(S_o - S_f)$$

- X - distance along the channel, t - time, v- average velocity, y - depth of flow, a- cross sectional area, w - top width, S_o - bed slope, S_f - friction slope

Saint Venant equations

- Friction slope

$$S_f = \frac{n^2 v^2}{r^{4/3}}$$

- r - hydraulic radius, n -Manning's roughness coefficient
- Two nonlinear equations in two unknowns v and y and two dependent variables x and t
- These two equations are a set of hyperbolic partial differential equations

Saint-Venant equations

- Multiplying 1st equation by $\pm \sqrt{g / \alpha w}$ and adding it to 2nd equation yields

$$\left[\frac{\partial}{\partial t} + (v \pm c) \frac{\partial}{\partial x} \right] v \pm \frac{1}{c} \left[\frac{\partial}{\partial t} + (v \pm c) \frac{\partial}{\partial x} \right] y = g(S_o - S_f)$$

- The above equation is a pair of equations along characteristics given by

$$\frac{dx}{dt} = v \pm c \quad \frac{dv}{dt} \pm \frac{g}{c} \frac{dy}{dt} = g(S_o - S_f)$$

- Based on the equations used, methods are classified as characteristics methods and direct methods.

FD methods for Saint Venant equations

- The governing equation in the conservation form may be written in matrix form as

$$U_t + F_x + S = 0$$

- In which

$$U = \begin{pmatrix} a \\ va \end{pmatrix}$$

$$F = \begin{pmatrix} va \\ v^2 a + ga\bar{y} \end{pmatrix}$$

$$S = \begin{pmatrix} 0 \\ -ga(s_0 - s_f) \end{pmatrix}$$

- General formulation

$$\frac{\partial f}{\partial t} = \frac{(f_i^{n+1} + f_{i+1}^{n+1}) - (f_i^n + f_{i+1}^n)}{\Delta t}$$

FD methods for Saint Venant equations

- Continued...

$$\frac{\partial f}{\partial x} = \frac{\alpha(f_{i+1}^{n+1} + f_i^{n+1})}{\Delta x} + \frac{(1-\alpha)(f_{i+1}^n + f_i^n)}{\Delta x}$$

$$f = \frac{1}{2}\alpha(f_{i+1}^{n+1} + f_i^{n+1}) + \frac{1}{2}(1-\alpha)(f_{i+1}^n + f_i^n)$$

$$\begin{aligned} U_i^{n+1} + U_{i+1}^{n+1} &= 2 \frac{\Delta t}{\Delta x} \left[\alpha(F_{i+1}^{n+1} - F_i^{n+1}) + (1-\alpha)(F_{i+1}^n - F_i^n) \right] \\ &\quad + \Delta t \left[\alpha(S_i^{n+1} + S_{i+1}^{n+1}) + (1-\alpha)(S_{i+1}^n + S_i^n) \right] \\ &= U_i^n + U_{i+1}^n \end{aligned}$$

FD methods for Saint Venant equations

- Boundary conditions:

$$y_{i,j+1}^{n+1} = y_{resd}$$

- Downstream boundary:

- Left boundary $y=y_u = \textit{uniform flow depth}$
 $v=v_u = \textit{uniform velocity}$
- Right boundary $y=y_c = \textit{Critical flow depth}$
 $v=v_c = \textit{Critical velocity}$

FD methods for Saint Venant equations

- Stability: unconditionally stable provided $\alpha > 0.5$, i.e., the flow variables are weighted toward the $n+1$ time level.
- Unconditional stability means that there is no restriction on the size of Δx and Δt for stability

Solution procedure

- The expansion of the equation...

$$a_i^{n+1} + a_{i+1}^{n+1} + 2 \frac{\Delta t}{\Delta x} \left\{ \alpha \left[(va)_{i+1}^{n+1} - (va)_i^{n+1} \right] + (1 - \alpha) \left[(va)_{i+1}^n - (va)_i^n \right] \right\}$$
$$= a_i^n + a_{i+1}^n$$

$$(va)_i^{n+1} + (va)_{i+1}^{n+1} + 2 \frac{\Delta t}{\Delta x} \left\{ \alpha \left[(v^2 a + ga\bar{y})_{i+1}^{n+1} - (v^2 a + ga\bar{y})_i^{n+1} \right] \right.$$
$$\left. - ga\Delta t \left\{ \alpha \left[(s_0 - s_f)_{i+1}^{n+1} + (s_0 - s_f)_i^{n+1} \right] \right\} \right.$$
$$= ga\Delta t \left\{ (1 - \alpha) \left[(s_0 - s_f)_{i+1}^n + (s_0 - s_f)_i^n \right] \right\}$$
$$+ (va)_i^n + (va)_{i+1}^n - (1 - \alpha)^2 \frac{\Delta t}{\Delta x} \left\{ (v^2 a + ga\bar{y})_{i+1}^n - (v^2 a + ga\bar{y})_i^n \right\}$$

- The above set of nonlinear algebraic equations can be solved by Newton-Raphson method

Assignments

1. Prove the following equation describes the gradually varied flow in a channel having variable cross section along its length:

$$\frac{dy}{dx} = \frac{S_o - S_f + (V^2 / gA) \partial A / \partial x}{1 - (BV^2) / (gA)}$$

2. Develop computer programs to compute the water- surface profile in a trapezoidal channel having a free overfall at the downstream end. To compute the profile, use the following methods:
 - (i) Euler method
 - (ii) Modified Euler method
 - (iii) Fourth-order Runge-Kutta method

Assignments

3. Using method of characteristics, write a computer program to solve 1D gradually varied unsteady flow in an open channel as given by Saint-Venant equations, assuming initial and boundary conditions.